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Optimizing and Analyzing the stability of The Rowat-Selverston Neural Oscillator for One-Legged Locomotion

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ABSTRACT

This study analyzes the mathematical model of Neural Oscillator, known as the Roth-Silverstone model, The main goal here to optimize and analyze the stability of the Roth-Silverstone neural. The paper also discusses how to generate new rhythmic patterns to produce gait generation rhythmic patterns for one leg with two degrees of freedoms (DOF) by using the genetic algorithm (GA) with a hybrid function. Finally, this paper will show new results compare with previous studies related to Central patterns generations (CPGs) without any feedback.

1 Introduction

Recent research has revealed that the fundamental movement patterns in biological organisms are orchestrated by a neural system known as central pattern generators (CPGs). The neural system is exited in the spinal cord in both vertebrate and invertebrate animals, Both the CPGs and neural oscillators are primarily situated within the spinal cord, and they can generate rhythmic patterns even in the absence of sensory input. Notably, certain bodily functions in humans, such as breathing and digestion as example, are autonomously regulated by these CPGs, beyond our conscious control[1], [2], [3] and [4].In the realm of robotics, numerous mathematical and physical models have been developed to mimic the movements of living creatures and reproduce rhythmic patterns akin to those observed in real biological systems see [5] [6] [7] [8]. This study is centred on the analysis and enhancement of central pattern generators, particularly in the context of controlling one leg movements with two degrees movements (DOFs). To achieve this, an optimization algorithm was applied to determine the most effective configuration for two central pattern generators.

The primary focus of the Rowat-Selverston model is on understanding the dynamics of rhythmic neural activity, such as the patterns of activity seen in the (CPGs) that control rhythmic movements in animals. These CPGs are crucial for generating coordinated movements like walking and swimming.

The Rowat-Selverston model is designed to capture the electrical properties and interactions of neurons within these neural circuits for more details see[9]. It takes into account various factors that influence neural oscillations, including the intrinsic properties of neurons (such as ion channel conductance), synaptic interactions, and network connectivity. Researchers use this model to explore how changes in parameters, such as the strengths of synaptic connections or the properties of individual neurons, can affect the generation and coordination of rhythmic neural activity.

By studying these simulations, scientists can gain insights into the underlying mechanisms of neural circuits and how they produce specific patterns of activity. The Rowat-Selverston Neural Oscillator Model is just one of many computational tools and models used in neuroscience to advance our understanding of neural systems and their behaviors. The main aim in this study focuses on optimizing the Rowat and Selverston Neural Oscillator Model to generate patterns to one leg with two degrees in freedoms in stable domain with different conditions.

The paper is organized as follows: The kinematic model has been discussed in the next section. A strategy to couple two Rowat and Selverston Neural Oscillators is given in Section 3. In Section 4, the stability analysis is discussed. Section 5 is devoted to the optimization results. In Section 6, some conclusions are drawn and suggestions for future research are given.

2 Kinematic Modelling of One Leg

One of the reasons is used the kinematic model that, it is used to analyze leg motion in two states: the first case is stance and the second case is swing. The simple motion equations in the kinematic model of the leg show the relationship between joint angles and the coordinates of critical points in the leg, namely $(x_f, y_f), (x_A, y_A)$ respectively.The first equation calculates the coordinates of the lower part of the hip joint (x_A, y_A) based on the length of the straight segment between the hip joint and the point (x_b, y_b) that represents the position of the hip in space.

$$x_A = x_b + L_1 cos\theta_1, \& y_A = y_b + L_1 sin\theta_1$$

Where L_1 is the length from the hip joint to the knee joint, and θ_1 is the angular position of the hip. The second equation calculates the coordinates of the lower part of the knee joint (x_f, y_f) based on the length of the straight segment between the hip joint and the knee joint and based on the angle of the knee joint.

$$x_f = x_A + L_2 cos\theta_2, \& \, y_f = y_A + L_2 sin\theta_2$$

Where L_2 is the length from the knee joint to the ankle joint, and θ_2 is the angular position of the knee. When the leg moves, it has two degrees of freedom. We consider in the first case leg with two degrees of freedom as seen in Figure 1.

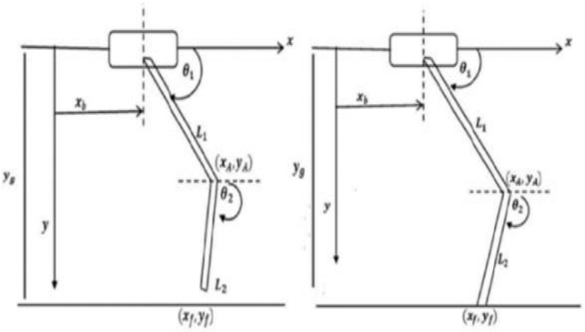

Figure 1: One leg in two cases stance and swing cases[10]

3 The Rowat and Selverston Neural Oscillator Model

Biological neurons with several ionic channels are complex, hence difficult to model. Rowat and Selverston (1993) present a simple model of a neuron for which two groups of currents are identified: a fast current and a slow current, each defined by a first order differential equation. Fast current is defined by Eq. 1 and slow current by Eq. 2.

$$\tau_m \left(\frac{dV}{dt}\right) = -F(V, \sigma_f) - q + I_{inj} \qquad (1)$$

$$\tau_s \left(\frac{dq}{dt}\right) = -q + \sigma_s V \qquad (2)$$

where,

$$F(V, \sigma_f) = V - A_f tan\, h\left(\frac{\sigma_f V}{A_f}\right)$$

Where τ_m and τ_s are the time constant of the neuron and the time constant of slow currents activation respectively. Where $\tau_m < \tau_s$, I_{inj} is the injected current, V the cellular membrane voltage, and q the slow current. $F(V, \sigma_f)$ is a non-linear current voltage for the fast current, and A_f adjusts the width and without affecting the degree of the N-shape. Difference Between RPA and Traditional Automation:Figure 2 shows structure of two Rowat-Selverston neural in Simulink block.

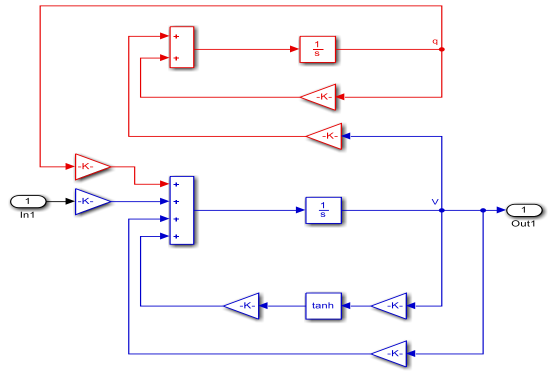

Figure 2: Internal Dynamics of two Rowat-Selverston neural (Uncoupled)

3 Stability Analysis

In this part, the stability analysis for each type of coupling given in the previous section will be discussed. We consider one neural as:

$$\left.\begin{aligned} \tau_m \left(\frac{dV}{dt}\right) &= -F(V, \sigma_f) - q + I_{inj} \\ \tau_s \left(\frac{dq}{dt}\right) &= -q + \sigma_s V \\ F(V, \sigma_f) &= V - A_f \tanh\left(\frac{\sigma_f V}{A_f}\right) \end{aligned}\right\} \qquad (3.1)$$

It is enough to find to analysis one to know the stabile region:

It is known that :

$$\tanh^{-1}(z) = \frac{1}{2} Log\left(\frac{1+z}{1-z}\right), |z| < 1\, also tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$
$$Log z = Ln(r) + i(\theta + 2n\pi) \quad , r > 0 \, , \alpha < \theta < \alpha + 2\pi$$

To find the equilibrium points of the given system (3.1), we need to find values of V and q such that both equations are satisfied:

$$\tau_m \left(\frac{dV}{dt}\right) = -F(V, \sigma_f) - q + I_{inj}$$

$$\tau_s \left(\frac{dq}{dt}\right) = -q + \sigma_s V$$

Let's first focus on the first equation:

$$\tau_m \left(\frac{dV}{dt}\right) = -F(V, \sigma_f) - q + I_{inj}$$

Rearranging the equation:

$$\tau_m\left(\frac{dV}{dt}\right) = -q - F\left(V, \sigma_f\right) + I_{inj}$$

Now, let's look at the second equation:

$$\tau_s\left(\frac{dq}{dt}\right) = -q + \sigma_s V$$

Rearranging this equation:

$$\tau_s\left(\frac{dq}{dt}\right) + q = \sigma_s V$$

Now, we want to find values of V and q such that both equations are satisfied simultaneously. This means that at the equilibrium points, both left-hand sides (LHS) and right-hand sides (RHS) of these equations must be equal. Therefore, we have. Equilibrium for the first equation:

$$\tau_m\left(\frac{dV}{dt}\right) = -q - F\left(V, \sigma_f\right) + I_{inj}$$

At equilibrium, $\frac{dV}{dt} = 0$, so we get:

$$0 = -q - F\left(V, \sigma_f\right) + I_{inj}$$

Equilibrium for the second equation:

$$\tau_s\left(\frac{dq}{dt}\right) + q = \sigma_s V$$

At equilibrium, $\frac{dq}{dt} = 0$, so we get:

$$q = \sigma_s V$$

Now, we have two equations:

$$0 = -q - F\left(V, \sigma_f\right) + I_{inj}$$

$$q = \sigma_s V$$

We can substitute the second equation into the first to find the equilibrium points:

$$0 = -\sigma_s V - F\left(V, \sigma_f\right) + I_{inj} \Rightarrow$$

$$0 = -\sigma_s V - \left(V - A_f \tanh\left(\frac{\sigma_f V}{A_f}\right)\right) + I_{inj}$$

Now, we need to solve this equation for V. The specific form of $F\left(V, \sigma_f\right)$ is not provided, so the equilibrium points will depend on the function $F\left(V, \sigma_f\right)$and the value of I_{inj}. You would need to know the details of $F\left(V, \sigma_f\right)$ to find the exact equilibrium points. Essentially, it is needed to solve this equation for V with the specific form of $F\left(V, \sigma_f\right)$ and the given value of I_{inj}.Keep in mind that this is a nonlinear equation, and the solutions may not have a simple closed-form expression. You may need to use numerical methods or software to find the equilibrium points numerically[5][5].

On solving the above equation using maple, we have the following result:

$$V = \frac{I_{inj}}{1 + \sigma_s}, \quad \sigma_f = \frac{in\pi(1 + \sigma_s)A_f}{I_{inj}} \quad , n \in I$$

Therefore:

$$q = \frac{\sigma_s I_{inj}}{1 + \sigma_s}$$

The first the equilibrium point is the origin point when we assume that $I_{inj} = 0$
In this case we will CPG instead of neural.When $I_{inj} \neq 0$

$$J\left(\frac{I_{inj}}{1+\sigma_s}, \frac{\sigma_s I_{inj}}{1+\sigma_s}\right) = \begin{bmatrix} \frac{1}{\tau_m}(-1-\sigma_f) & -\frac{1}{\tau_m} \\ \frac{\sigma_s}{\tau_s} & -\frac{1}{\tau_s} \end{bmatrix}$$

Eigenvalues are:

$$\lambda_{1,2} = \frac{\left(-\tau_s\sigma_f - \tau_m - \tau_s \pm \sqrt{\tau_s^2\sigma_f^2 - 2\tau_m\tau_s\sigma_f - 4\tau_m\tau_s\sigma_s + 2\tau_s^2\sigma_f + \tau_m^2 - 2\tau_m\tau_s + \tau_s^2}\right)}{2\tau_s\tau_m}$$

If $\sigma_f < 1$ while σ_s is large enough, the cell does not oscillate as seen in the figure 3-a

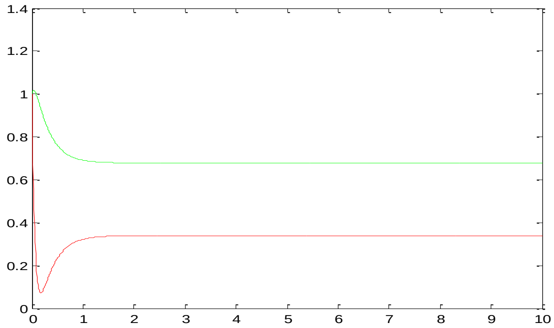

Figure3-a: two Rowat-Selverston neural when $\sigma_f = 0.05$ while $\sigma_s = 2$

if $\sigma_f > 1$ but σ_s is reduced below a certain value, then cell shows plateau potentials as seen in the figure 3-b

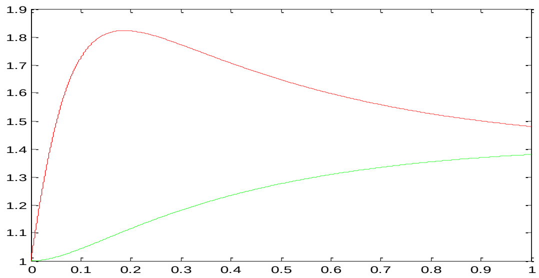

Figure3-b: two Rowat-Selverston neural when $\sigma_f = 2.2$ while $\sigma_s = 1$

If we increase the value of σ_s above one, then the cell starts to oscillate as seen in the figure 3-c

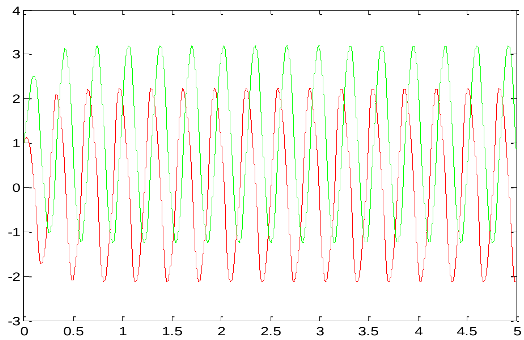

Figure3-c: two Rowat-Selverston neural when $\sigma_f = 2$ while $\sigma_s = 20$

If we increase the value of σ_s for large and decrease the value of σ_f, then we will obtain damped oscillation as seen in the figure 3-d

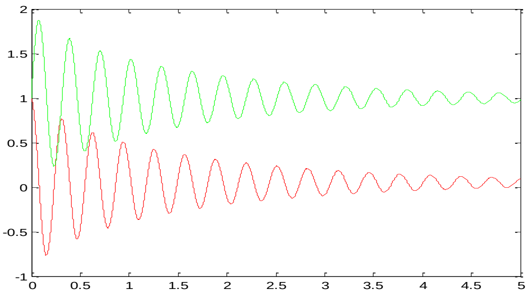

Figure3-d: two Rowat-Selverston neural when $\sigma_f = 1$ while $\sigma_s = 20$

Note that, if $\sigma_f < 1$ while σs is large enough, the cell does not oscillate, if $\sigma_f < 1$but σ_s is reducedbelow a certainvalue, then cell shows plateau potentials , If we increase the value of σ_s above one,then the cell starts to oscillate, Ifwe increase the value of σ_s for large and decrease the value of σ_f,then we will obtain damped oscillation.

4 Optimization results

It is important to mention here that each harmonic oscillator of a couple produces angular patterns as outputs for each joint. For estimating gait generation, it is very important to compute the optimal parameter sets for each harmonic oscillator. Of course, it is necessary to know how the angular position of both joints will change in time for generating rhythmic patterns. The parameter sets in the case . The (GA) will be used to find the values of the parameters that optimize the objective function given below. The different walking patterns rely on this cost function:

$$J = -c_1\sum_{k=1}^{n} x_b(k) + c_2\left(\sum_{k=1}^{n}\left(\theta_1^2(k) + \theta_2^2(k)\right)\right)/N$$

The variables C_1, C_2 are used to illustrate the relationship between velocity and energy constraints during the improvement of CPG, $C_1, C_2 \in [0,1]$ and $C_1 + C_2 = 1$, n is the number of position vector elements, and N is the length of time. In this study, it is assumed that $C_1 \neq 0$. Furthermore, the requirements $C_1 + C_2 = 1$ are used to reduce power consumption with position change, actually this study is used both case $C_1 + C_2 = 1$ and $C_1 + C_2 = 2$ to make balance with position change.To maximize the displacement, or the velocity, we should minimize J. If $C_2 = 0$, then the aim is to maximize the displacement. However, if $C_1C_2 \neq 0$, then there will be another cost function involving energy related terms in addition to the position. The goal is to minimize energy while changing the position. Actually, this fact is available in biological locomotion.

The two constraints revealed here are $0 \leq \theta_1, \theta_2 \leq \pi$ due to physical reasons. In the present study, a hybrid function is used during the optimization process which runs after the GA terminates for improving the value of the fitness function. The final point from GA is used as the initial point in either Pattern Search (PS) or fmin search (FM). We have performed an optimization process using the ODE45 and ODE23 methods, with varying population sizes in each method to select the appropriate population size for solving this problem. Additionally, we have changed the crossover from 0.2 to 1 each time to merge the genetic characteristics of different solutions and generate new solutions with improved characteristics through the hybridization and mutation process. We obtained results that are collectively located in the stability region, as shown in the table1.

Tables 1: Uncoupled two neural oscillators in bounded in 20 seconds (N_in_pop=45*)*

Uncoupled two oscillators neural in bounded region (N_in_pop=45) by solve rode 45			
Parameters value $\{\sigma_{s1}, \sigma_{f1}, \tau_{s1}, \tau_{m1}, A_{f1}, I_{inj1}, \sigma_{s2}, \sigma_{f2}, \tau_{s2}, \tau_{m2}, A_{f2}, I_{inj2}$	x_b	J	E
2.26954.04320.17840.87744.72384.7309 0.96045.46630.87442.66354.45725.1739	0.5605	-100.1109	634.1651
2.26994.0423 0.17890.8779 4.7241 4.7323 0.9604 5.4662 0.8746 2.6645 4.4574 5.1745	0.5611	-100.3596	634.0800
2.5126 4.4630 0.4556 0.9076 4.9547 4.8410 1.0756 4.1377 1.0705 1.0464 4.7013 5.3884	0.7949	-441.1785	614.2520
2.4984 4.4588 0.4567 0.9164 4.9558 4.8344 1.0767 4.1398 1.0707 1.0464 4.7040 5.3885	0.7977	-441.7518	617.3701
2.5004 4.4594 0.4567 0.9166 4.9536 4.8348 1.0767 4.1401 1.0697 1.0464 4.7040 5.3895	0.7985	-442.5758	617.0095
2.5159 4.4745 0.4570 0.9167 4.9526 5.1366 1.0767 4.1438 1.0707 1.0491 4.7040 5.3896	0.7931	-437.3793	615.5156
2.5160 4.4746 0.4570 0.9167 4.9526 5.1366 1.0758 4.1303 1.0715 1.0457 4.7025 5.3898	0.7923	-436.8146	615.1885

Some the corresponding figures relate to table 1 are shown in Figures 3, 4, 5 and 6

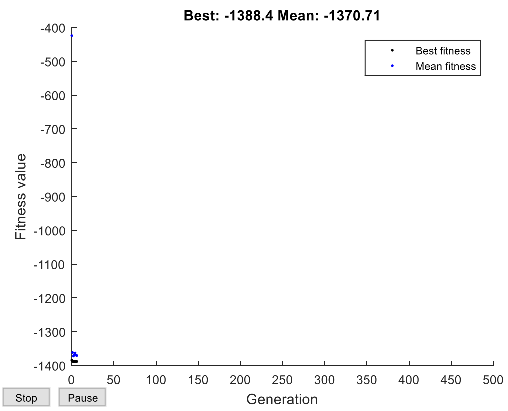

Figure 4: The optimization results are corresponding to the initial conditions $V(0) = 1$ and $q(0) = 1$,with crossover 0.8.

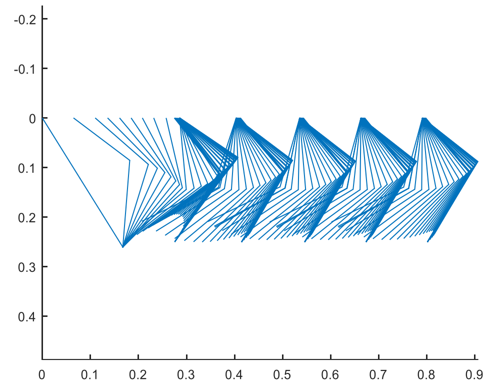

Figure5: One leg animation Uncoupled two oscillators neural: This animation corresponds to the values :$\sigma_{s1} = 2.5004 \sigma_{f1} = 4.4594 \tau_{s1} = 0.4567 \tau_{m1} = 0.9166 A_{f1} = 4.9536\ I_{inj1} = 4.8348 \sigma_{s2} = 1.0767 \sigma_{f2} = 4.1401 \tau_{s1} = 1.0697 \tau_{m1} = 1.0464 A_{f2} = 4.7040 I_{inj2} = 5.3895$

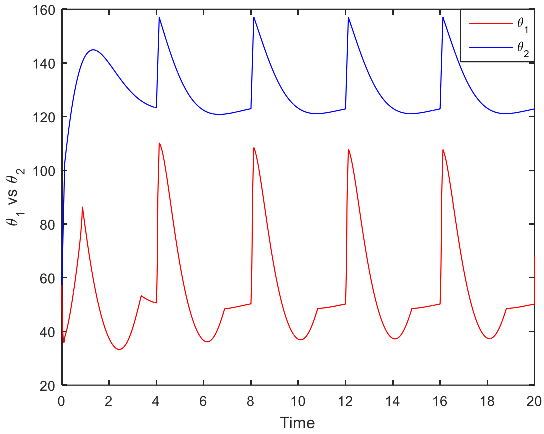

Figure6: The Outputs of the oscillators neural: This solution corresponds to the same values in Fig4

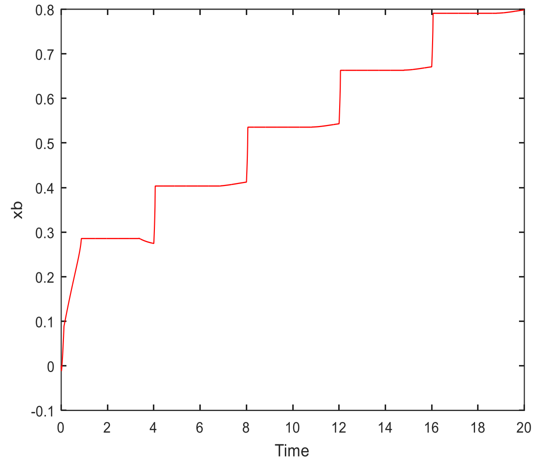

Figure7: displacement against time for Uncoupled two oscillators neural: This solution corresponds to the same values in theFig4.

Tables 2: Uncoupled two Uncoupled two neural oscillators in bounded region in 20 seconds (N_in_pop=120)

Uncoupled two oscillatorsneural in bounded region**in 20 second(N_in_pop=120) by ode23**			
Parameters value $\{\sigma_{s1},\sigma_{f1},\tau_{s1},\tau_{m1},A_{f1},I_{inj1},\sigma_{s2},\sigma_{f2},\tau_{s2},\tau_{m2},A_{f2},I_{inj2}\}$	X_b	J	E
2.1522 4.3387 0.5797 0.8160 3.8589 5.2892 1.1099 5.0216 1.6163 1.0431 4.5553 5.6377	0.8412	- 493.3451	594.171 6
2.1569 4.3439 0.5788 0.8176 3.8663 5.2933 1.1106 5.0238 1.6150 1.0442 4.5577 5.6379	0.8410	- 493.2107	594.324 6
2.5267 4.5483 0.3270 0.8504 3.6386 5.6252 0.3613 4.7133 1.9473 0.0234 4.1052 5.8369	1.8128	- 964.3638	1141.9
2.5283 4.5480 0.3264 0.8502 3.6389 5.6295 0.3614 4.7409 1.9551 0.0234 4.1053 5.8370	1.8132	- 964.7322	1141.8
0.7285 1.6954 0.4564 0.8502 4.5080 5.6295 0.3614 4.7409 1.9551 0.0234 4.1053 5.8370	1.8117	- 894.1402	1208.4

Some the corresponding figures relate to table 1 are shown in Figures 7, 8, 9 and 10

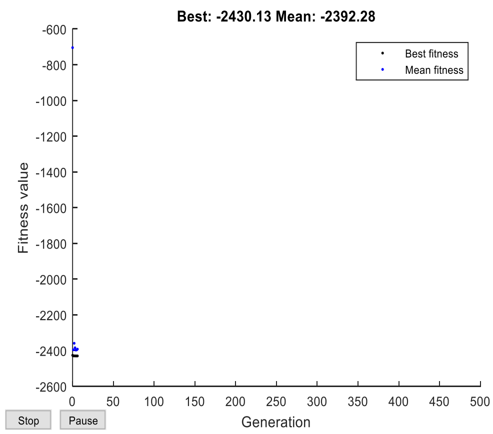

Figure8: The optimization results are corresponding to the initial conditions $V(0) = 1$ and $q(0) = 1$,with crossover 0.8

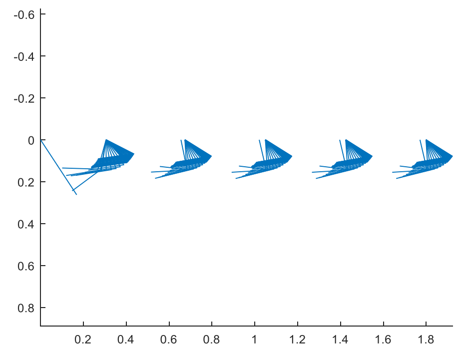

Figure9: One leg animation Uncoupled two oscillators neural: This animation corresponds to the values :

$\sigma_{s1} = 2.5283$, $\sigma_{f1} = 4.5480$, $\tau_{s1} = 0.3264$, $\tau_{m1} = 0.8502$, $A_{f1} = 3.6389$, $I_{inj1} = 5.6295$, $\sigma_{s2} = 0.3614$, $\sigma_{f2} = 4.7409$, $\tau_{s2} = 1.9551, \tau_{m2} = 0.0234$, $A_{f2} = 4.1053$, $I\_inj2 = 5.8370$

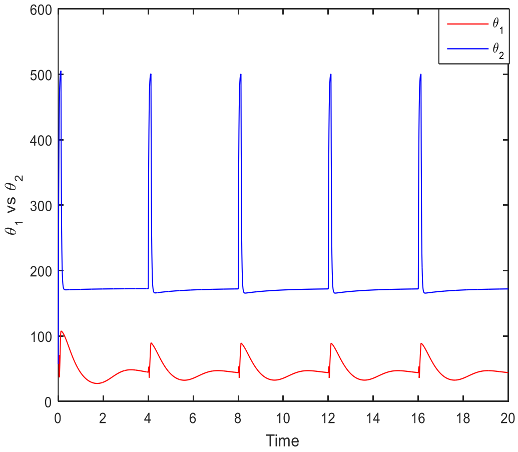

Figure10: The Outputs of the oscillators neural: This solution corresponds to the same values in the Fig8

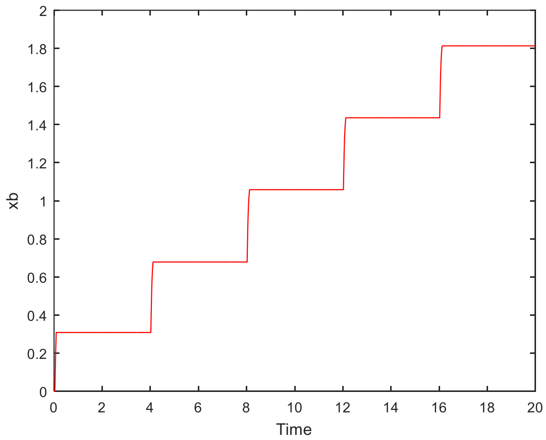

Figure11: displacement against time for Uncoupled two oscillators neural: This solution corresponds to the same values in the Fig8.

Based on the results shown in the tables above, we observe that $\tau_s < \tau_m$ and the difference was small. The results were not affected and remained stable region. During the optimization online we use two solvers, the first one isode45, the results show that the distance is normal the best the choose for population size is 45 and crossover is 0.8.

While in the second trial, when the optimization has done population size 60, we obtained that the best result is, as results of that when the crossover is 0.8, the distance is 0.7600.

Thirdly, when the optimization is done under population size 90, as seen that the best result is obtained, in addition to the crossover is0.4, we get that the distance is 0.7732.

Finally remarkable, when the optimization is done under population size 120, we found that the best result, the crossover is0.2 we obtain distance is 0.7721. Our choice for time 20 second and solver 45 ode respectively for all the previous results.

From the results of the previous results, it is noted that the best result is conducted when the optimization is done when the population size is 45 and the crossover is 0.8. in this case the distance is 0.7985 by using ode 45.

While the optimization is done by using the ode23 solver and under population size is 45, also the crossover is 0.2. We obtain distance is 0.5135, we use here ode 23 solver.

While in the second trial, when we conducted the optimization under population size 60, we found that the best result when the crossover is0.8, we obtain the distance is 0.7926 by using ode 23.

The third trial, the optimization is under population size 90, we found that the best result when the crossover was 0.4 we obtain distance is 0.8029 by using ode 23 in Simulink block.

Likewise in the fourth trial, the optimization is done under population size 120, we found that the best result when the crossover was 0.8 we obtain distance is 1.8132, we used ode 23 in Simulink block.

From the results of the previous trials, itis observed that the best result is during the fourth trial under the population size 120, when the crossover is 0.8 we obtained distance is 1.8132.

5 Conclusion and Future Directions

To sum up, in this paper two Rowat and Selverston Neural Oscillator sun coupled are used to generate motion for one leg with two degrees of freedom. The study shows that when optimization is conducted in an bounded region and stable domain, the rhythmic pattern is able to move leg with two degree of freedoms normal as seen in natural. When we consider the stability analysis presented above with the objective of decreasing the variation between steps, the optimization achieved easily local minimum values of the parameters of neural oscillators model. It is vital that we control the amplitude and the frequency to obtain better results. Such results, we believe, can be implemented physically**.** Most important, the study reveals Rowat and Selverston Neural Oscillators can control biped locomotion not only in animals but also in human beings. Future research should investigate whether Rowat and Selverston Neural Oscillators can control other functions in human bodies, such as breathing, let alone the stimulation of the arm movement.

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